Chapter 27
Sire Model, Multiple Traits

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1984 - Guelph

1 Only One Trait Observed On A Progeny

This section deals with a rather simple model in which there are t traits measured on the progeny of a set of sires. But the design is such that only one trait is measured on any progeny. This results in $\mathbf{R}$ being diagonal. It is assumed that each dam has only one recorded progeny, and the dams are non-inbred and unrelated. An additive genetic model is assumed. Order the observations by progeny within traits. There are t traits and k sires. Then the model is

$$\begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_t \end{pmatrix} = \begin{pmatrix} \mathbf{X}_1 & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_2 & \ldots & \mathbf{0} \\ \vdots & \vdots & & \vdots \\ \mathbf{0} & \mathbf{0} & & \mathbf{X}_t \end{pmatrix} \begin{pmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \\ \vdots \\ \boldsymbol{\beta}_t \end{pmatrix} + \begin{pmatrix} \mathbf{Z}_1 & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{Z}_2 & \ldots & \mathbf{0} \\ \vdots & \vdots & & \vdots \\ \mathbf{0} & \mathbf{0} & & \mathbf{Z}_t \end{pmatrix} \begin{pmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \\ \vdots \\ \mathbf{s}_t \end{pmatrix} + \begin{pmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \vdots \\ \mathbf{e}_t \end{pmatrix} \quad (1)$$

$\mathbf{y}_i$ represents n_i progeny records on trait i, $\boldsymbol{\beta}_i$ is the vector of fixed effects influencing the records on the i^{th} trait, $\mathbf{X}_i$ relates $\boldsymbol{\beta}_i$ to elements of $\mathbf{y}_i$, and $\mathbf{s}_i$ is the vector of sire effects for the i^{th} trait. It has k has a null column corresponding to such a sire.

$$Var \begin{pmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \\ \vdots \\ \mathbf{s}_t \end{pmatrix} = \begin{pmatrix} \mathbf{A}b_{11} & \mathbf{A}b_{12} & \ldots & \mathbf{A}b_{1t} \\ \mathbf{A}b_{12} & \mathbf{A}b_{22} & \ldots & \mathbf{A}b_{2t} \\ \vdots & \vdots & & \vdots \\ \mathbf{A}b_{1t} & \mathbf{A}b_{2t} & \ldots & \mathbf{A}b_{tt} \end{pmatrix} = \mathbf{G}. \quad (2)$$

$\mathbf{A}$ is the $k \times k$ numerator relationship matrix for the sires. If the sires were unselected, $b_{ij} = g_{ij}/4$, where g_{ij} is the additive genetic covariance between traits i and j.

$$Var \begin{pmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \vdots \\ \mathbf{e}_t \end{pmatrix} = \begin{pmatrix} \mathbf{I}d_1 & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{I}d_2 & \ldots & \mathbf{0} \\ \vdots & \vdots & & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & \mathbf{I}d_t \end{pmatrix} = \mathbf{R}. \quad (3)$$

Under the assumption of unselected sires

$$d_i = .75\ g_{ii} + r_{ii},$$

where r_{ii} is the i^{th} diagonal of the error covariance matrix of the usual multiple trait model. Then the GLS equations for fixed $\mathbf{s}$ are

$$\begin{pmatrix} d_1^{-1}\mathbf{X'}_1\mathbf{X}_1 & \ldots & \mathbf{0} & d_1^{-1}\mathbf{X'}_1\mathbf{Z}_1 & \ldots & \mathbf{0} \\ \vdots & & \vdots & \vdots & & \vdots \\ \mathbf{0} & \ldots & d_t^{-1}\mathbf{X'}_t\mathbf{X}_t & \mathbf{0} & \ldots & d_t^{-1}\mathbf{X'}_t\mathbf{Z}_t \\ d_1^{-1}\mathbf{Z'}_1\mathbf{X}_1 & \ldots & \mathbf{0} & d_1^{-1}\mathbf{Z'}_1\mathbf{Z}_1 & \ldots & \mathbf{0} \\ \vdots & & \vdots & \vdots & & \vdots \\ \mathbf{0} & \ldots & d_t^{-1}\mathbf{Z'}_t\mathbf{X}_t & \mathbf{0} & \ldots & d_t^{-1}\mathbf{Z'}_t\mathbf{Z}_t \end{pmatrix}$$

$$\begin{pmatrix} \boldsymbol{\beta}_1^o \\ \vdots \\ \boldsymbol{\beta}_t^o \\ \hat{\mathbf{s}}_1 \\ \vdots \\ \hat{\mathbf{s}}_t \end{pmatrix} = \begin{pmatrix} d_1^{-1}\mathbf{X'}_1\mathbf{y}_1 \\ \vdots \\ d_t^{-1}\mathbf{X'}_t\mathbf{y}_t \\ d_1^{-1}\mathbf{Z'}_1\mathbf{y}_1 \\ \vdots \\ d_1^{-1}\mathbf{Z'}_t\mathbf{y}_t \end{pmatrix} \quad (4)$$

The mixed model equations are formed by adding $\mathbf{G}^{-1}$ to the lower right $(kt)^2$ submatrix of (27.4), where

$$\mathbf{G}^{-1} = \begin{pmatrix} \mathbf{A}^{-1}b^{11} & \ldots & \mathbf{A}^{-1}b^{1t} \\ \vdots & & \vdots \\ \mathbf{A}^{-1}b^{1t} & \ldots & \mathbf{A}^{-1}b^{tt} \end{pmatrix}, \quad (5)$$

and b^{ij} is the ij^{th} element of the inverse of

$$\begin{pmatrix} b_{11} & \ldots & b_{1t} \\ \vdots & & \vdots \\ b_{1t} & \ldots & b_{tt} \end{pmatrix}.$$

With this model it seems logical to estimate d_i by

$$[\mathbf{y'}_i\mathbf{y}_i - (\boldsymbol{\beta}_i^o)'\mathbf{X'}_i\mathbf{y}_i - (\mathbf{u}_i^o)'\mathbf{Z'}_i\mathbf{y}_i]/[n_i - \text{rank}(\mathbf{X}_i\ \ \mathbf{Z}_i)]. \quad (6)$$

$\boldsymbol{\beta}_i^o$ and $\mathbf{u}_i^o$ are some solution to (27.7)

$$\begin{pmatrix} \mathbf{X'}_i\mathbf{X}_i & \mathbf{X'}_i\mathbf{Z}_i \\ \mathbf{Z'}_i\mathbf{X}_i & \mathbf{Z'}_i\mathbf{Z}_i \end{pmatrix} \begin{pmatrix} \boldsymbol{\beta}_i^o \\ \mathbf{u}_i^o \end{pmatrix} = \begin{pmatrix} \mathbf{X'}_i\mathbf{y}_i \\ \mathbf{Z'}_i\mathbf{y}_i \end{pmatrix}. \quad (7)$$

Then using these $\hat{d}_i$, estimate the b_{ij} by quadratics in $\hat{\mathbf{s}}$, the solution to (27.4). The quadratics needed are

$$\hat{\mathbf{s}}_i'\mathbf{A}^{-1}\hat{\mathbf{s}}_j; \quad i = 1, \ldots, t; \quad j = i, \ldots, t.$$

These are computed and equated to their expectations. We illustrate this section with a small example. The observations on progeny of three sires and two traits are

Sire	Trait	Progeny Records
1	1	5,3,6
2	1	7,4
3	1	5,3,8,6
1	2	5,7
2	2	9,8,6,5

Suppose $\mathbf{X}_1' = [1 \ldots 1]$ with 9 elements, and $\mathbf{X}_2' = [1 \ldots 1]$ with 6 elements.

$$\mathbf{Z}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{Z}_2 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

Suppose that

$$\mathbf{R} = \begin{pmatrix} 30\,\mathbf{I}_9 & \mathbf{0} \\ \mathbf{0} & 25\,\mathbf{I}_6 \end{pmatrix},$$

$$\mathbf{A} = \begin{pmatrix} 1. & .5 & .5 \\ .5 & 1. & .25 \\ .5 & .25 & 1. \end{pmatrix},$$

and

$$\begin{pmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}.$$

Then

$$\mathbf{G} = \begin{pmatrix} 3. & 1.5 & 1.5 & 1. & .5 & .5 \\ & 3. & .75 & .5 & 1. & .25 \\ & & 3. & .5 & .25 & 1. \\ & & & 2. & 1. & 1. \\ & & & & 2. & .5 \\ & & & & & 2. \end{pmatrix},$$

$$\mathbf{G}^{-1} = \begin{pmatrix} 10 & -4 & -4 & -5 & 2 & 2 \\ & 8 & 0 & 2 & -4 & 0 \\ & & 8 & 2 & 0 & -4 \\ & & & 15 & -6 & -6 \\ & & & & 12 & 0 \\ & & & & & 12 \end{pmatrix} \frac{1}{15}.$$

$$\begin{pmatrix} \mathbf{X}'\mathbf{R}^{-1} \\ \mathbf{Z}'\mathbf{R}^{-1} \end{pmatrix} (\mathbf{X} \quad \mathbf{Z}) = \frac{1}{150} \begin{pmatrix} 45 & 0 & 15 & 10 & 20 & 0 & 0 & 0 \\ & 36 & 0 & 0 & 0 & 12 & 24 & 0 \\ & & 15 & 0 & 0 & 0 & 0 & 0 \\ & & & 10 & 0 & 0 & 0 & 0 \\ & & & & 20 & 0 & 0 & 0 \\ & & & & & 12 & 0 & 0 \\ & & & & & & 24 & 0 \\ & & & & & & & 0 \end{pmatrix} \tag{8}$$

Adding $\mathbf{G}^{-1}$ to the lower 6×6 submatrix of (27.8) gives the mixed model coefficient matrix. The right hand sides are [1.5667, 1.6, .4667, .3667, .7333, .48, 1.12, 0]. The inverse of the mixed model coefficient matrix is

$$\begin{pmatrix} 5.210 & 0.566 & -1.981 & -1.545 & -1.964 & -0.654 & -0.521 & -0.652 \\ 0.566 & 5.706 & -0.660 & -0.785 & -0.384 & -1.344 & -1.638 & -0.690 \\ -1.981 & -0.660 & 2.858 & 1.515 & 1.556 & 0.934 & 0.523 & 0.510 \\ -1.545 & -0.785 & 1.515 & 2.803 & 0.939 & 0.522 & 0.917 & 0.322 \\ -1.964 & -0.384 & 1.556 & 0.939 & 2.783 & 0.510 & 0.322 & 0.923 \\ -0.654 & -1.344 & 0.934 & 0.522 & 0.510 & 1.939 & 1.047 & 0.984 \\ -0.521 & -1.638 & 0.523 & 0.917 & 0.322 & 1.047 & 1.933 & 0.544 \\ -0.652 & -0.690 & 0.510 & 0.322 & 0.923 & 0.984 & 0.544 & 1.965 \end{pmatrix} \tag{9}$$

The solution to the MME is (5.2380, 6.6589, -.0950, .0236, .0239, -.0709, .0471, -.0116).

2 Multiple Traits Recorded On A Progeny

When multiple traits are observed on individual progeny, $\mathbf{R}$ is no longer diagonal. The linear model can still be written as (27.1). Now, however, the $\mathbf{y}_i$ do not have the same number of elements, and $\mathbf{X}_i$ and $\mathbf{Z}_i$ have varying numbers of rows. Further,

$$\mathbf{R} = \begin{pmatrix} \mathbf{I} \ r_{11} & \mathbf{P}_{12} r_{12} & \ldots & \mathbf{P}_{1t} r_{1t} \\ \mathbf{P}'_{12} r_{12} & \mathbf{I} \ r_{22} & \ldots & \mathbf{P}_{2t} r_{2t} \\ \vdots & \vdots & & \vdots \\ \mathbf{P}'_{1t} r_{1t} & \mathbf{P}'_{2t} r_{2t} & \ldots & \mathbf{I} \ r_{tt} \end{pmatrix}. \tag{10}$$

The $\mathbf{I}$ matrices have order equal to the number of progeny with that trait recorded.

$$\begin{pmatrix} r_{11} & \ldots & r_{1t} \\ \vdots & & \vdots \\ r_{1t} & \ldots & r_{tt} \end{pmatrix}$$

is the error variance-covariance matrix. We can use the same strategy as in Chapter 25 for missing data. That is, each $\mathbf{y}_i$ is the same length with 0's inserted for missing data.

Accordingly, all $\mathbf{X}_i$ and $\mathbf{Z}_i$ have the same number of rows with rows pertaining to missing observations set to 0. Further, $\mathbf{R}$ is the same as for no missing data except that rows corresponding to missing observations are set to 0. Then the zeroed type of g-inverse of $\mathbf{R}$ is

$$\begin{pmatrix} \mathbf{D}_{11} & \mathbf{D}_{12} & \ldots & \mathbf{D}_{1t} \\ \mathbf{D}_{12} & \mathbf{D}_{22} & \ldots & \mathbf{D}_{2t} \\ \vdots & \vdots & & \vdots \\ \mathbf{D}_{1t} & \mathbf{D}_{2t} & \ldots & \mathbf{D}_{tt} \end{pmatrix}. \quad (11)$$

Each of the $\mathbf{D}_{ij}$ is diagonal with order, n. Now the GLS equations for fixed $\mathbf{s}$ are

$$\begin{pmatrix} \mathbf{X'}_1\mathbf{D}_{11}\mathbf{X}_1 & \ldots & \mathbf{X'}_1\mathbf{D}_{1t}\mathbf{X}_t & \mathbf{X'}_1\mathbf{D}_{11}\mathbf{Z}_1 & \ldots & \mathbf{X'}_1\mathbf{D}_{1t}\mathbf{Z}_t \\ \vdots & & \vdots & \vdots & & \vdots \\ \mathbf{X'}_t\mathbf{D}_{1t}\mathbf{X}_1 & \ldots & \mathbf{X'}_t\mathbf{D}_{tt}\mathbf{X}_t & \mathbf{X'}_t\mathbf{D}_{1t}\mathbf{Z}_1 & \ldots & \mathbf{X'}_t\mathbf{D}_{tt}\mathbf{Z}_t \\ \mathbf{Z'}_1\mathbf{D}_{11}\mathbf{X}_1 & \ldots & \mathbf{Z'}_1\mathbf{D}_{1t}\mathbf{X}_t & \mathbf{Z'}_1\mathbf{D}_{11}\mathbf{Z}_1 & \ldots & \mathbf{Z'}_1\mathbf{D}_{1t}\mathbf{Z}_t \\ \vdots & & \vdots & \vdots & & \vdots \\ \mathbf{Z'}_t\mathbf{D}_{1t}\mathbf{X}_1 & \ldots & \mathbf{Z'}_t\mathbf{D}_{tt}\mathbf{X}_t & \mathbf{Z'}_t\mathbf{D}_{1t}\mathbf{Z}_1 & \ldots & \mathbf{Z'}_t\mathbf{D}_{tt}\mathbf{Z}_t \end{pmatrix} \begin{pmatrix} \boldsymbol{\beta}_1^o \\ \vdots \\ \boldsymbol{\beta}_t^o \\ \hat{s}_1 \\ \vdots \\ \hat{s}_t \end{pmatrix} =$$

$$\begin{pmatrix} \mathbf{X'}_1\mathbf{D}_{11}\mathbf{y}_1 & + & \ldots & + & \mathbf{X'}_1\mathbf{D}_{1t}\mathbf{y}_t \\ \vdots & & & & \vdots \\ \mathbf{X'}_t\mathbf{D}_{1t}\mathbf{y}_1 & + & \ldots & + & \mathbf{X'}_t\mathbf{D}_{tt}\mathbf{y}_t \\ \mathbf{Z'}_1\mathbf{D}_{11}\mathbf{y}_1 & + & \ldots & + & \mathbf{Z'}_1\mathbf{D}_{1t}\mathbf{y}_t \\ \vdots & & & & \vdots \\ \mathbf{Z'}_t\mathbf{D}_{1t}\mathbf{y}_1 & + & \ldots & + & \mathbf{Z'}_t\mathbf{D}_{tt}\mathbf{y}_t \end{pmatrix}. \quad (12)$$

With $\mathbf{G}^{-1}$ added to the lower part of (27.12) we have the mixed model equations.

We illustrate with the following example.

		Trait	
Sire	Progeny	1	2
1	1	6	5
	2	3	5
	3	-	7
	4	8	-
2	5	4	6
	6	-	7
	7	3	-
3	8	5	4
	9	8	-

We assume the same $\mathbf{G}$ as in the illustration of Section 27.1, and

$$\begin{pmatrix} r_{11} & r_{12} \\ r_{12} & r_{22} \end{pmatrix} = \begin{pmatrix} 30 & 10 \\ 10 & 25 \end{pmatrix}.$$

We assume that the only fixed effects are μ_1 and μ_2 . Then using the data vector with length 13, ordered progeny in sire in trait,

$$\mathbf{X}_1' = (1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1), \quad \mathbf{X}_2' = (1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1),$$

$$\mathbf{Z}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{Z}_2 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\mathbf{y}_1' = (6\ \ 3\ \ 8\ \ 4\ \ 3\ \ 5\ \ 8), \quad \mathbf{y}_2' = (5\ \ 5\ \ 7\ \ 6\ \ 7\ \ 4),$$

and

$$\mathbf{R} = \begin{pmatrix} 30 & 0 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 & 0 \\ & & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & 0 \\ & & & & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 10 \\ & & & & & & 30 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & 25 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & & 25 & 0 & 0 & 0 & 0 \\ & & & & & & & & & 25 & 0 & 0 & 0 \\ & & & & & & & & & & 25 & 0 & 0 \\ & & & & & & & & & & & 25 & 0 \\ & & & & & & & & & & & & 25 \end{pmatrix}.$$

Then the GLS coefficient matrix for fixed **s** is in (27.13).

$$\begin{pmatrix} 0.254 & -0.061 & 0.110 & 0.072 & 0.072 & -0.031 & -0.015 & -0.015 \\ -0.061 & 0.265 & -0.031 & -0.015 & -0.015 & 0.132 & 0.086 & 0.046 \\ 0.110 & -0.031 & 0.110 & 0.0 & 0.0 & -0.031 & 0.0 & 0.0 \\ 0.072 & -0.015 & 0.0 & 0.072 & 0.0 & 0.0 & -0.015 & 0.0 \\ 0.071 & -0.015 & 0.0 & 0.0 & 0.072 & 0.0 & 0.0 & -0.015 \\ -0.031 & 0.132 & -0.031 & 0.0 & 0.0 & 0.132 & 0.0 & 0.0 \\ -0.015 & 0.086 & 0.0 & -0.015 & 0.0 & 0.0 & 0.086 & 0.0 \\ -0.015 & 0.046 & 0.0 & 0.0 & -0.015 & 0.0 & 0.0 & 0.046 \end{pmatrix} \tag{13}$$

$\mathbf{G}^{-1}$ is added to the lower 6×6 submatrix to form the mixed model coefficient matrix. The right hand sides are (1.0179, 1.2062, .4590, .1615, .3974, .6031, .4954, .1077). The inverse of the coefficient matrix is

$$\begin{pmatrix} 6.065 & 1.607 & -2.111 & -1.735 & -1.709 & -0.702 & -0.603 & -0.546 \\ 1.607 & 5.323 & -0.711 & -0.625 & -0.519 & -1.472 & -1.246 & -1.017 \\ -2.111 & -0.711 & 2.880 & 1.533 & 1.527 & 0.953 & 0.517 & 0.506 \\ -1.735 & -0.625 & 1.533 & 2.841 & 0.893 & 0.517 & 0.938 & 0.303 \\ -1.709 & -0.519 & 1.527 & 0.893 & 2.844 & 0.506 & 0.303 & 0.944 \\ -0.702 & -1.472 & 0.953 & 0.517 & 0.506 & 1.939 & 1.028 & 1.003 \\ -0.602 & -1.246 & 0.517 & 0.938 & 0.303 & 1.028 & 1.924 & 0.562 \\ -0.546 & -1.017 & 0.506 & 0.303 & 0.943 & 1.002 & 0.562 & 1.936 \end{pmatrix}. \quad (14)$$

The solution is [5.4038, 5.8080, .0547, -.1941, .1668, .0184, .0264, -.0356].

If we use the technique of including in $\mathbf{y}$, $\mathbf{X}$, $\mathbf{Z}$, $\mathbf{R}$, $\mathbf{G}$ the missing data we have

$$\mathbf{X}_1' = (1\ \ 1\ \ 0\ \ 1\ \ 1\ \ 0\ \ 1\ \ 1\ \ 1), \quad \mathbf{X}_2' = (1\ \ 1\ \ 1\ \ 0\ \ 1\ \ 1\ \ 0\ \ 1\ \ 0),$$

$$\mathbf{Z}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{Z}_2 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$

$$\mathbf{y}_1' = (6,\ 3,\ 0,\ 8,\ 4,\ 0,\ 3,\ 5,\ 8),$$

and

$$\mathbf{y}_2' = (5,\ 5,\ 7,\ 0,\ 6,\ 7,\ 0,\ 4,\ 0).$$

$$\begin{aligned} \mathbf{D}_{11} &= \text{diag}[.0385,\ .0385,\ 0,\ .0333,\ .0385,\ 0,\ .0333,\ .0385,\ .0333] \\ \mathbf{D}_{12} &= \text{diag}[-.0154,\ -.0154,\ 0,\ 0,\ -.0154,\ 0,\ 0,\ -.0154,\ 0] \\ \mathbf{D}_{22} &= \text{diag}[.0462,\ .0462,\ .04,\ 0,\ .0462,\ .04,\ 0,\ .0462,\ 0]. \end{aligned}$$

This leads to the same set of equations and solution as when $\mathbf{y}$ has 13 elements.

3 Relationship To Sire Model With Repeated Records On Progeny

The methods of Section 27.2 could be used for sire evaluation using progeny with repeated records (lactations, e.g.), but we do not wish to invoke the simple repeatability model. Then lactation 1 is trait 1, lactation 2 is trait 2, etc.