Chapter 25
Sire Model, Repeated Records

C. R. Henderson

1984 - Guelph

This chapter is a combination of those of Chapters 23 and 24. That is, we are concerned with progeny testing of sires, but some progeny have more than one record. The scalar model is

$$y_{ijk} \;=\; \mathbf{x}'_{ijk}\boldsymbol{\beta} \;+\; \mathbf{z}'_{ijk}\mathbf{u} \;+\; s_i \;+\; p_{ij} \;+\; e_{ijk}.$$

$\mathbf{u}$ represents random factors other than $\mathbf{s}$ and $\mathbf{p}$. It is assumed that all dams are unrelated and all progeny are non-inbred. Under an additive genetic model the covariance between any record on one progeny and any record on another progeny of the same sire is $\sigma^2_s = \frac{1}{4}\ h^2\ \sigma^2_y$ if sires are a random sample from the population. The covariance between any pair of records on the same progeny is $\sigma^2_s + \sigma^2_p = r\sigma^2_y$. If sires are unselected, $\sigma^2_p = (r - \frac{1}{4}h^2)\sigma^2_y$, $\sigma^2_e = (1-r)\sigma^2_y$, $\sigma^2_s = \frac{1}{4}h^2\sigma^2_y$.

In vector notation the model is

$$\begin{aligned} \mathbf{y} \;&=\; \mathbf{X}\boldsymbol{\beta} \;+\; \mathbf{Z}\mathbf{u} \;+\; \mathbf{Z}_s\mathbf{s} \;+\; \mathbf{Z}_p\mathbf{p} \;+\; \mathbf{e}. \\ Var(\mathbf{s}) \;&=\; \mathbf{A}\ \sigma^2_s,\ Var(\mathbf{p}) \;=\; \mathbf{I}\ \sigma^2_p,\ Var(\mathbf{e}) \;=\; \mathbf{I}\ \sigma^2_e. \end{aligned}$$

With field data one might eliminate progeny that do not have a first record in order to reduce bias due to culling, which is usually more intense on first than on later records. Further, if a cow changes herds, the records only in the first herd might be used. In this case useful computing strategies can be employed. The data can be entered by herds, and $\mathbf{p}$ easily absorbed because $\mathbf{Z}'_p\mathbf{Z}_p \;+\; \mathbf{I}\sigma^2_e/\sigma^2_p$ is diagonal. Once this has been done, fixed effects pertinent to that particular herd can be absorbed. These methods are described in detail in Ufford *et al*. (1979). They are illustrated also in a simple example which follows.

We have a model in which the fixed effects are herd-years. The observations are displayed in the following table.

		Herd - years						
Sires	Progeny	11	12	13	21	22	23	24
1	1	5	6	4				
	2	5	8	-				
	3	-	9	4				
	4				5	6	7	3
	5				4	5	-	-
	6				-	4	3	-
	7				-	-	2	8
2	8	7	6	-				
	9	-	5	4				
	10	-	9	-				
	11	-	-	4				
	12				3	7	6	-
	13				-	5	6	8
	14				-	-	5	4

We assume $\sigma_e^2/\sigma_s^2 = 8.8$, $\sigma_e^2/\sigma_p^2 = 1.41935$. These correspond to unselected sires, $h^2 = .25$, r $= .45$. Further, we assume that **A** for the 2 sires is

$$\begin{pmatrix} 1 & .25 \\ .25 & 1 \end{pmatrix}.$$

Ordering the solution vector **hy**, **s**, **p** the matrix of coefficients of OLS equations is in (25.1), and the right hand side vector is (17, 43, 16, 12, 27, 29, 23, 88, 79, 15, 13, 13, 21, 9, 7, 10, 13, 9, 9, 4, 16, 19, 9)$'$.

$$\begin{aligned}
\mathbf{X}'\mathbf{X} &= \text{diag } (3, 6, 4, 3, 5, 6, 4) \\
\mathbf{Z}_s'\mathbf{Z}_s &= \text{diag } (17, 14) \\
\mathbf{Z}_p'\mathbf{Z}_p &= \text{diag } (3, 2, 2, 4, 2, 2, 2, 2, 2, 1, 1, 3, 3, 2) \\
\mathbf{Z}_s'\mathbf{X} &= \begin{pmatrix} 2 & 3 & 2 & 2 & 3 & 3 & 2 \\ 1 & 3 & 2 & 1 & 2 & 3 & 2 \end{pmatrix} \\
\mathbf{Z}_s'\mathbf{Z}_p &= \begin{pmatrix} 3 & 2 & 2 & 4 & 2 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 1 & 1 & 3 & 3 & 2 \end{pmatrix}
\end{aligned}$$

$$\mathbf{Z}_p'\mathbf{X} = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \tag{1}$$

Modifying these by adding 8.8 $\mathbf{A}^{-1}$ and 1.41935 $\mathbf{I}$ to appropriate submatrices of the coefficient matrix, the BLUP solution is

$$\begin{aligned} \widehat{\mathbf{hy}} &= [5.83397, 7.14937, 4.18706, 3.76589, 5.29825, 4.83644, 5.64274]', \\ \hat{\mathbf{s}} &= [-.06397, .06397]', \\ \hat{\mathbf{p}} &= [-.44769, .04229, .52394, .31601, .01866, -.87933, -.10272, \\ &\quad -.03255, -.72072, .73848, -.10376, .43162, .68577, -.47001]'. \end{aligned}$$

If one absorbs $\mathbf{p}$ in the mixed model equations, we obtain

$$\begin{pmatrix} 2.189 & -.811 & -.226 & 0 & 0 & 0 & 0 & .736 & .415 \\ & 4.191 & -.811 & 0 & 0 & 0 & 0 & 1.151 & 1.417 \\ & & 2.775 & 0 & 0 & 0 & 0 & .736 & 1.002 \\ & & & 2.297 & -.703 & -.411 & -.184 & .677 & .321 \\ & & & & 3.778 & -.930 & -.411 & 1.092 & .642 \\ & & & & & 4.486 & -.996 & 1.092 & 1.057 \\ & & & & & & 3.004 & .677 & .736 \\ & & & & & & & 15.549 & -2.347 \\ & & & & & & & & 14.978 \end{pmatrix}$$

$$\begin{pmatrix} \widehat{\mathbf{hy}} \\ \hat{\mathbf{s}} \end{pmatrix} = \begin{pmatrix} 6.002 \\ 21.848 \\ 4.518 \\ 1.872 \\ 10.526 \\ 9.602 \\ 9.269 \\ 31.902 \\ 31.735 \end{pmatrix}.$$

The solution for $\widehat{\mathbf{hy}}$ and $\hat{\mathbf{s}}$ are the same as before.

If one chooses, and this would be mandatory in large sets of data, $\widehat{\mathbf{hy}}$ can be absorbed herd by herd. Note that the coefficients of $\widehat{\mathbf{hy}}$ are in block diagonal form. When $\widehat{\mathbf{hy}}$ is absorbed, the equations obtained are

$$\begin{pmatrix} 12.26353 & -5.222353 \\ -5.22353 & 12.26353 \end{pmatrix} \begin{pmatrix} \hat{\mathbf{s}}_1 \\ \hat{\mathbf{s}}_2 \end{pmatrix} = \begin{pmatrix} -1.1870 \\ 1.1870 \end{pmatrix}.$$

The solution is approximately the same for sires as before, the approximation being due to rounding errors.